Inecuaciones. Ejercicios

1 Resolver las siguientes inecuaciones

1 $2(x+1)-3(x-2)<x+6$

2 $\frac{3x+1}{7}-\frac{2-4x}{3} \geq \frac{-5x-4}{14}+\frac{7x}{6}$

3 $6\left(\frac{x+1}{8}-\frac{2x-3}{16}\right) > 3\left(\frac{3}{4}x-\frac{1}{4}\right)-\frac{3}{8}(3x-2)$

2 Resolver las inecuaciones:

1 $7x^2+21x-28<0$

2 $-x^2+4x-7<0$

3 $4x^2-16 \geq 0$

3 Resuelve:

1 $x^4+12x^3-64x^2>0$

2 $x^4-25x^2-144<0$

3 $x^4-16x^2-225 \geq 0$

4 Resolver las inecuaciones:

1 $\frac{x^2-1}{-x^2+2x-1} \leq 0$

2 $\frac{x^2-1}{x^2-4} \leq 0$

SOLUCIONES

1

Resolver las siguientes inecuaciones

1 $$2(x+1)-3(x-2)<x+6$$

$$2x+2-3x+6<x+6$$

$$4x-6x+6x<-12+40$$

$$4x<28 \qquad\qquad x<7$$

(-∞ 7)

2 $$\frac{3x+1}{7}-\frac{2-4x}{3}\geq\frac{-5x-4}{14}+\frac{7x}{6}$$

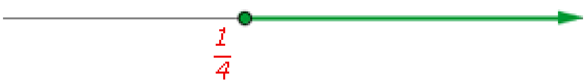
$$m.c.m.(7,\ 3,\ 14,\ 6)=42$$

$$6(3x+1)-14(2-4x)\geq 3(-5x-4)+49x$$

$$18x+6-28+56x\geq -15x-12+49x$$

$$18x+56x+15x-49x\geq -12-6+28$$

$$\mathbf{40x\geq 10} \qquad\qquad \mathbf{4x\geq 1} \qquad\qquad x\geq\frac{1}{4}$$



$$\left[\frac{1}{4},\ \infty\right)$$

3 $$6\left(\frac{x+1}{8}-\frac{2x-3}{16}\right)>3\left(\frac{3}{4}x-\frac{1}{4}\right)-\frac{3}{8}(3x-2)$$

$$\frac{6(x+1)}{8}-\frac{6(2x-3)}{16}>\frac{9}{4}x-\frac{3}{4}-\frac{9}{8}x+\frac{6}{8}$$

$$\text{m.c.m.}(8,\ 16,\ 4)=16$$

$$\cancel{12x}+12\cancel{-12x}+18>36x\cancel{-12}-18x\cancel{+12}$$

$$12+18>36x-18x$$

$$18x<30 \qquad 3x<5 \qquad\qquad x<\frac{5}{3}$$

$$\left(-\infty,\ \frac{5}{3}\right)$$

2

Resolver las inecuaciones:

1 $7x^2 + 21x - 28 < 0$ dividimos por 3

$x^2 + 3x - 4 < 0$

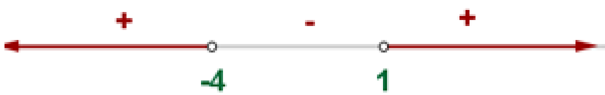
$x^2 + 3x - 4 = 0$

$$x=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm 5}{2}= \begin{cases} x_1 = 1 \\ x_2 = -4 \end{cases}$$

$P(-6) = (-6)^2 + 3 \cdot (-6) - 4 > 0$

$P(0) = 0^2 + 3 \cdot 0 - 4 < 0$

$P(3) = 3^2 + 3 \cdot 3 - 4 > 0$



$(-4,\ 1)$

2 $-x^2 + 4x - 7 < 0$

$x^2 - 4x + 7 = 0$

$$x = \frac{4 \pm \sqrt{16-28}}{2} = \frac{-3 \pm \sqrt{-12}}{2} \notin \mathbb{R}$$

$P(0) = -0^2 + 4 \cdot 0 - 7 < 0$

$S = \mathbb{R}$

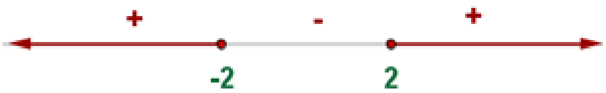
3 $4x^2 - 16 \geq 0$

$4x^2 = 16$ $x^2 = 4$ $x = \pm\sqrt{4}$ $\nearrow x_1 = 2$ $\searrow x_2 = -2$

$P(-3) = 4 \cdot (-3)^2 - 16 > 0$

$P(0) = 4 \cdot 0^2 - 16 < 0$

$P(3) = 4 \cdot 3^2 - 16 > 0$



$(-\infty, -2] \cup [2, +\infty)$

3

Resuelve:

1 $x^4 + 12x^3 - 64x^2 > 0$ sacamos factor común x

$x^2(x^2 + 12x - 64) > 0$

Como el primer factor es siempre positivo, sólo tendremos que estudiar el signo del 2º factor.

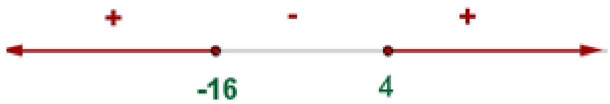
$x^2 + 12x - 64 = 0$

$$x = \frac{-12 \pm \sqrt{144+256}}{2} = \frac{-12 \pm 20}{2} = \begin{cases} x_2 = 4 \\ x_3 = -16 \end{cases}$$

$P(-17) = (-17)^2 + 12 \cdot 17 - 64 > 0$

$P(0) = 0^2 + 12 \cdot 0 - 64 < 0$

$P(5) = 5^2 + 12 \cdot 5 - 64 > 0$



$(-\infty, -16] \cup [4, \infty)$

2 $\quad x^4 - 25x^2 - 144 < 0$

$x^4 - 25x^2 - 144 = 0$

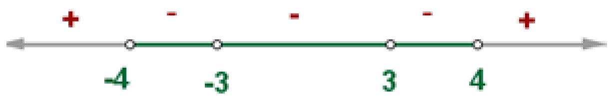
$x^2 = t$

$t^2 - 25t - 144 = 0$

$$t = \frac{25 \pm \sqrt{625 - 576}}{2} = \frac{25 \pm 7}{2} = \begin{cases} t_1 = 16 \\ t_2 = 9 \end{cases}$$

$x^2 = 16 \qquad x = \pm\sqrt{16} \begin{cases} x_1 = 4 \\ x_2 = -4 \end{cases}$

$x^2 = 9 \qquad x = \pm\sqrt{9} \begin{cases} x_1 = 3 \\ x_2 = -3 \end{cases}$



$(-4, -3) \cup (-3, 3) \cup (3, 4)$.

3 $x^4 - 16x^2 - 225 \geq 0$

$x^4 - 16x^2 - 225 = 0$

$x^2 = t$

$t^2 - 16t - 225 = 0$

$$t = \frac{16 \pm \sqrt{256+900}}{2} = \frac{16 \pm 34}{2} = \begin{cases} t_1 = 25 \\ t_2 = -9 \end{cases}$$

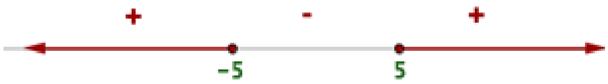
$$x^2 = 25 \qquad x = \pm\sqrt{25} \begin{cases} x_1 = 5 \\ x_2 = -5 \end{cases}$$

$$x^2 = -9 \qquad x = \pm\sqrt{-9} \notin \mathbb{R}$$

$(x^2 - 25) \cdot (x^2 + 9) \geq 0$

El segundo factor siempre es positivo y distinto de cero, sólo tenemos que estudiar el signo del 1er factor.

$(x^2 - 25) \geq 0$



$(-\infty, -5] \cup [5, +\infty)$

4

Resolver las inecuaciones:

1 $$\frac{x^2 - 1}{-x^2 + 2x - 1} \leq 0$$

$x^2 - 1 = 0 \qquad x = \pm 1$

$-x^2 + 2x - 1 = 0 \qquad x^2 - 2x + 1 = 0 \qquad x = 1$

$-(x-1)^2$

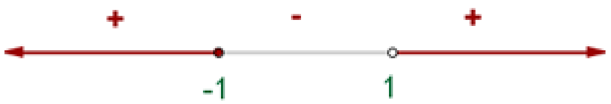
$$\frac{x^2-1}{-\underset{\substack{\downarrow \\ +}}{\boxed{(x-1)^2}}} \leq 0 \qquad\qquad x \neq 1$$

El binomio elevado al cuadrado es siempre positivo, pero al tener delante el signo menos. resultará que el demnominador será siempre negativo.

$$\frac{x^2-1}{-1} \leq 0 \qquad\qquad x \neq 1$$

Multiplicando por −1:

$$x^2-1 \geq 0 \qquad\qquad x \neq 1$$



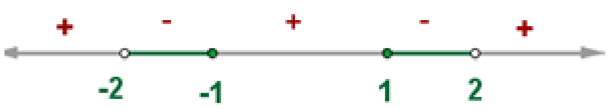
$(-\infty, -1] \cup (1, +\infty)$

2 $\qquad \dfrac{x^2-1}{x^2-4} \leq 0$

$x^2-1=0 \qquad\qquad x=\pm 1$

$x^2-4=0 \qquad\qquad x=\pm 2$

$$\frac{x^2-1}{x^2-4} \leq 0 \qquad\qquad x \neq \pm 2$$



$[-2, -1] \cup (1, 2)$